ON A NECESSARY AND SUFFICIENT CRITERION FOR BRITTLE STRENGTH

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The criterion (2.1) is proposed as a necessary condition to estimate the strength of an elastic body weakened by a slit. It is shown that the resulting formula for the critical crack length agrees, in practice, with the Griffith result. The concept of considering crack-slits in elastic bodies (surfaces of normal displacement jumps) as nontrivial equilibrium states of a physically nonlinear elastic medium is given a foundation. It is shown that the theory of crack [1] substantially examines the problem in precisely such a formulation, by linearizing it by the energy balance method. It is remarked that the crack stability criterion obtained by this means is not only necessary, but also sufficient. However, it turns out that the necessary and sufficient criteria yield practically coincident results in the problem under consideration.

1. Let us consider the well known problem of elasticity theory: tension of a plane with an elliptical cutout by normal stresses $\sigma_{yy}(x, \pm\infty) = \sigma$ acting perpendicularly to the major axis of the ellipse.

In this case the maximum normal stresses originate at the points $x = {}^1/_2 l$. When utilizing the linearized equations of classical elasticity theory they are given by

$$\max(\sigma_{yy}) = \sigma\left[1 + \left(\frac{2l}{\rho}\right)^{1/2}\right] \qquad \left(\rho = \frac{l^2}{2h}\right) \tag{1.1}$$

where ρ is the least radius of curvature of the ellipse, l, h are its major and minor axes.

It is customary to consider the inequality

$$\sigma_{\max} \geqslant \sigma_c \tag{1.2}$$

as the brittle fracture condition.

Here $\sigma_{\max}$ is the greatest positive normal stress in the body, and σ_c is the limiting rupture stress.

Applying this criterion to the problem under consideration, we arrive at the conclusion that for given stresses σ at infinity and a radius of curvature ρ the inequality

$$l < l_* = \frac{(\sigma_c - \sigma)^2}{2\sigma^2}\rho \tag{1.3}$$

will be the strength condition for a body weakened by an elliptical crack, where l_* is the critical crack length. Putting $\rho = 0$ in (1.3), i.e. passing to the case of a crack in the shape of a slit, we obtain

$$l_* = 0 \tag{1.4}$$

Therefore, the deduction that the presence of a slit crack should induce the fracture of the body for arbitrarily small finite stresses σ, no matter how small the crack length, follows from linear elasticity theory and the criterion (1.2). However, Griffith [1] showed from energy considerations that fracture should occur in the case of a slit crack (analogously to the case of an elliptical crack with finite transverse dimension) only if the crack length exceeds some critical value. Griffith proposed the inequality

$$dU > dR \tag{1.5}$$

as the energy criterion for the fracture of a body with a slit, where dU is the decrease in elastic energy of the body because of a dl increase in the crack (for $\sigma \neq$ const), and dR is the work which must be expended to rupture the body by dl. When (1.5) is valid, the crack expands without limit, and is in the critical state in the case

$$dU = dR \tag{1.6}$$

For the problem of tension of a plane weakened by a slit oriented across the tension direction, linear theory of elasticity yields

$$dU = 2\pi l \frac{\sigma^2}{E} dl \tag{1.7}$$

As regards dR, Griffith assumed that the work required to rupture the body per unit area is a constant 2γ, a physical constant characterizing the rupture strength of the material. In the enegry formulation of the problem this constant will be the analog of the constant σ_c utilized in solving the stress problem. In the particualr case under consideration

$$dR = 2dl \tag{1.8}$$

Substituting (1.8) into (1.7) we arrive at the formula

$$l_g = \frac{4E\gamma}{\pi\sigma^2} = \frac{A}{\sigma^2}, \quad A = \frac{4E\gamma}{\pi} \tag{1.9}$$

The constant A depends on the mechanical properties of the body material.

Therefore, two different, and apparently both correct, methods of reasoning based on the same theoretical solution of the problem of the stress distribution in a slit crack yield essentially dissimilar results. The application of the more general criterion (1.2), which is valid for a hole of any shape, hence results in the particular case under consideration in an explicitly unequally likely deduction, not verified by tests, while the particular criterion (1.8), concocted especially for slit cracks yields an equally likely result verified by tests. A. Griffith himself noted the correct means of interpreting this paradox, and later Elliot then developed it in more detail [3].

The fact is that in the neighborhood of the ends of the slit not only the stresses, but also their gradients, are infinite according to linear elasticity theory, consequently it is impossible to neglect the change in stress in the areas of their maximum values even at distances on the order of the atomic radius. This idea is developed below.

2. The fracture of solids is a discrete process ; for example, it is impossible to separate half an atom from half an atom retaining the connection between their remaining halves. The destruction of the connection between just one pair of atoms will be a fracture "quantum". Here and henceforth, keeping in mind the roughness of the subsequent reasoning which will rely on the apparatus of linear elasticity theory, it is meaningless to devote oneself to the peculiarities of atomic lattice structure. The lattices are treated as a set of adjoining atomic layers, where the atoms of each successive layer are located above the atoms of the preceding layer, i. e. ideal (not densely packed) cubic lattices are examined.

If account is taken of the above, the fracture criterion (1.2) in the domain of high stress gradients should be written in the integral form

$$\max\left(\int_0^{2a} \sigma_{yy}\, dx\right) \geqslant 2a\sigma_c \tag{2.1}$$

where σ_{yy} is the normal stress perpendicular to the atomic layer, x is the spacing measured along a rectilinear chain of atoms in the direction of most rapid change in stress, and a is the atomic radius.

The necessity of the criterion (2.1) is evident. If the given inequality is not satisfied, fracture cannot occur since the external forces applied to the body turn out not to be in a state to overcome the maximum value of the intratomic forces even for one pair of atoms. Let us show that application of this criterion to the problem of tension of a plane weakened by a slit crack reduces to a formula practically coincident with the Griffith result (1.9). The distribution of normal stresses σ_{yy} along the x-axis near the end of the crack is defined by the asymptotic formula [4]

$$\sigma_{yy} = \sigma\left(1 + \frac{1}{2}\frac{l^{1/2}}{x^{1/2}}\right) \tag{2.2}$$

The y-axis is here perpendicular to the crack, and the x-axis is directed along the crack from the origin to the end being considered. On the basis of (2.2)

$$\int_0^{2a} \sigma_{yy}\, dx = 2a\sigma\left[1 + \left(\frac{l}{2a}\right)^{1/2}\right] \tag{2.3}$$

Substituting (2.3) into (2.1) we obtain

$$\sigma_c - \sigma = \sigma\left(\frac{l_*}{2a}\right)^{1/2}, \quad \text{or} \quad l_* = \frac{2(\sigma_c - \sigma)^2}{\sigma^2}a \tag{2.4}$$

According to existing estimates [5]

$$1/_4\, E \geqslant \sigma_c \geqslant 1/_{13}\, E \tag{2.5}$$

Here σ_c is the rupture yield point of a defectless atomic lattice. Hence, in the majority of cases it is permissible to neglect σ in (2.5) as compared with σ_c, after which (the quantity B is a physical constant)

$$l_* \approx \frac{2a\sigma_c^2}{\sigma^2} = \frac{B}{\sigma^2}, \qquad B = 2a\sigma_c^2 \tag{2.6}$$

In its form (2.6) agrees with the Griffith formula (1.9). Let us show that the constants A, B in these formulas are sufficiently alike. Indeed, according to (1.9) and (2.6)

$$\frac{A}{B} = \frac{2}{\pi}\frac{E\gamma}{a\sigma_c^2} \approx \frac{2}{\pi} \tag{2.7}$$

Since according to the estimate of E. Orowan [7]

$$\gamma \approx \frac{\sigma_c^2 a}{E} \tag{2.8}$$

Taking account of the approximateness of (2.8) (it has been derived under the assumption that the dependence of the force of interaction between two atoms on the change in the spacing between them is approximated by half a sinusoid), it can be said that (1.9) and (2.6) practically coincide.

It is hence seen that the Griffith formula has been derived not only by energy means, but also from the stress strength criterion under the natural extension of this criterion to the case of high stress gradients.

3. As is known, the stress distribution in the neighborhood of a slit crack can be obtained by passing to the limit from the solution of the problem of the stress distribution in the neighborhood of an elliptical crack for $\rho \to 0$. Formula (2.6) should hence also be obtained by an analogous passage to the limit from the formula for the critical length of an elliptical crack if the strength criterion is used in the "discrete form" (2.1) rather than the "point" form (1.2) in its derivation. It is quite interesting to execute a corresponding refinement of the result (1.3). According to G. V. Kolosov formulas, the stress

distribution along the x-axis is

$$\sigma_{yy} = 2\varphi'(x) + x\varphi''(x) + \psi'(x) \tag{3.1}$$

Hence

$$\int \sigma_{yy}\, dx = \varphi(x) + \psi(x) + x\varphi'(x) + \text{const} \tag{3.2}$$

In the problem of the extension of a plane with an elliptical cutout [6]

$$\varphi(x) = \frac{1}{2}\sigma\left[x + C\left(\frac{1}{\xi} - m\xi\right)\right], \quad \psi(x) = -C\sigma(1+m^2)\frac{\xi}{1-m\xi^2} \tag{3.3}$$

$$\xi = \frac{1}{2m_1C}\left[x - \sqrt{x^2 - {}^1/_4(l^2 - h^2)}\right], \quad C = \frac{l+h}{2}, \quad m_1 = \frac{l-h}{l+h}$$

Substituting (3.3) into (3.2), assuming therefore that $a / l \ll 1,\ h / l \ll 1$, and integrating between the limits ${}^1/_2 \ll x \ll {}^1/_2\, l + 2a$, we have

$$\int_{{}^1/_2 l}^{{}^1/_2 l + 2a} \sigma_{yy}\, dx \approx 2a\sigma\left[1 + \left(\frac{2l}{\rho + 4a}\right)^{1/2}\right] \tag{3.4}$$

After substituting (3.4) into (2.1) we arrive at the following formula for the critical length of an elliptical crack:

$$l_* \approx \frac{(\sigma_c - \sigma)^2}{2\sigma^2}(\rho + 4a) \tag{3.5}$$

This formula is more exact than (1.3) since discreteness of the fracture process is taken into account in (3.5).

For sufficiently narrow cracks $\sigma \ll \sigma_c$, approximately

$$l_* \approx \frac{\sigma_c^2}{2\sigma^2}(\rho + 4a) \tag{3.6}$$

Hence, (1.3) is obtained from (3.5) for $a \ll \rho$, and we have (2.6) for $\rho \ll a$, as should have been expected. The preceptive value of (3.6) is that it permits interpretation of the Griffith formula not only as a valid result for slit cracks, but also as an approximate solution for "solid" cracks with sufficiently small radii of curvature of their endpoints, where the error estimate is seen in passing from (3.6) to (2.6).

Meanwhile, in order to reconcile the Griffith formula to the stress strength criterion, Cottrell [8] postulated that (1.1) can be utilized only for $\rho \geqslant \rho^*$, and σ_{max} should be considered independent of ρ and equal to $\sigma_{max}(\rho^*)$ for $\rho < \rho^*$. Cottrell determined the magnitude of the effective radius of curvature of the end ρ^* of the crack by demanding that for a crack with $\rho < \rho^*$ (including slit cracks) the critical crack length should coincide with its value according to Griffith. It hence turns out that $\rho^* \approx 3a$. The result (3.5) obtained by means of other considerations justifies the Cottrell assumption. In fact, (3.5), being bases on the generalized strength criterion (2.2), can be obtained formally from (1.3) derived by starting from the "point" strength criterion, by replacing the radius of curvature ρ of the end of the crack in this latter by

$$\rho^* = \rho + 4\,a \tag{3.7}$$

where

$$\rho^* = 4a \tag{3.8}$$

for a slit crack, which practically agrees with the Cottrell recommendation.

4. Relative to the derivation proposed above for the Griffith formula from the stress strength criterion, the objection can be advanced that continuum mechanics has no right to consider dimensions on the order of an atomic diameter as a finite quantity.

However, this objection refers equally to the energy theory of Griffith, as well as to other existing theories which consider cracks as slits, since they, at least in veiled form, contain the atomic radius as a characteristic parameter of the problem. Indeed, according to (2.6) and (2.8)

$$\gamma = \alpha E a \tag{4.1}$$

Here α is a dimensionless coefficient. Its mean value is usually taken as

$$\alpha \approx 0.1 \tag{4.2}$$

Dividing γ by E we obtain the constant of the length dimension

$$\beta = \gamma / E \approx 0.1a \tag{4.3}$$

Therefore, even in the Griffith theory, altough in latent form, the physical constant alien to continuum mechanics, the atomic radius, is contained. It is also present in the δ_* of the crack theory of Leonov and Panasiuk [1, 9 and 10], where the equality

$$2\ \gamma = \sigma_c \delta_* \tag{4.4}$$

plays an essential part.

In the case of ideal brittle fracture, γ and σ_c in (4.4) are determined by (4.1) and (2.6). Taking (4.2) as the mean value for α and putting $\sigma_c \approx 0.1E$, we obtain

$$\delta_* \approx a \tag{4.5}$$

i.e. in this case δ_* practically coincides with the atomic radius.

The appearance of this physical constant in all brittle fracture theories which consider cracks as ideal slits, is inevitable since it is necessary to have at least one characteristic value of the length dimension in addition to the crack length in such a formulation of the problem. It is neither in the conditions of the problem nor in the classical theory of elasticity, and it can only be introduced by taking account of the discreteness of the structure of solids. The single characteristic dimension in ideal atomic lattices is the radius of the atom, hence it is just this that enters the theory of slit cracks as the physical constant supplementing the constants of elasticity theory. There should be added also that even the problem of quasi-brittle fracture (i.e. the question of slit crack propagation in elastoplastic solids) cannot be solved without involving the physical constant of the length dimension. Since there are no such constants in either elasticity or plasticity theories, it should be sought among the characteristics associated with the discreteness of the structure of solids (grain size, mean spacing between lattice defects, atomic radius). But this subject is not embraced in the present study.

5. As has been remarked, the criterion (2.1) is necessary. If the inequality (2.1) is not satisfied, then fracture may be known not to occur since in this case the stresses are insufficient for the maximum value of the interaction force to be exceeded for at least one pair of atoms.

For small stress gradients, when the change in these latter can be neglected within the limits of many atomic series, the criterion (2.1) will be not only a necessary, but a sufficient condition for macroscopic fracture of a body. But in the case of large stress gradients (as occur in problems of the stress concentration near slots and slits, say), the sufficiency of the criterion (2.1) becomes doubtful since it assures overcoming of the maximum interaction force between just two atomic series. Upon being included among other atomic series retaining stability, these two series turn out to be constrained in their displacements, and their failure should seemingly not involve macroscopic fracture of the body. The situation recalls that known from the theory of the carrying capacity of statically indeterminate systems for which the buckling or fracture of individual elements still does not mean fracture of the system as a whole.

It follows from the above that upon being applied to the problem of slit cracks, the criterion (2. 1) should yield a lower bound for the fracturing stress σ, and correspondingly for the critical crack length. A more exact estimate of the strength of a body weakened by a slot requires an investigation of the stresses in this problem taking into account the true picture of atom interaction in the neighborhood of the ends of the slot rather than on the basis of Hooke's law.

6. Let $T = 2a\sigma$ denote the interaction force between two arrays of atoms referred to unit length. The dependence of T on the change in spacing η between the arrays series is as shown in Fig. 1.

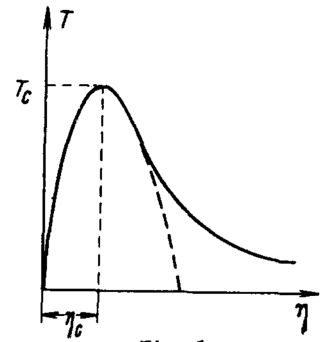

Fig. 1

One of the appropriate mathematical forms of this dependence will be

$$T = 2a\sigma = E\eta e^{-\eta/\eta_c} \tag{6.1}$$

where η_c is the value of η corresponding to $T_{max} = 2a\sigma_c$, where

$$T_{max} = E\eta_c e^{-1}, \qquad \sigma_c = \frac{E\eta_c}{2a} e^{-1} \tag{6.2}$$

There are ascending ($\eta \leqslant \eta_c$) and descending ($\eta > \eta_c$) portions of the curve $T \sim \eta$, where there is an essential difference in the nature of the deformation depending on whether the stress is on the former or latter portion.

The stresses grow on the first portion as the deformation increases (divergence between the atomic arrays), and conversely, decrease on the second. The equilibrium of the atoms is stable at each point of the ascending portion, and is unstable at each point of the descending portion since in this latter case the atoms start to diverge without limit for a small deviation from the equilibrium position towards a growth in deformation, while the magnitude of the stress is conserved.

Let us consider the following situation (Fig. 2): let the spacing between two plane atomic layers, whose position is considered fixed, be $2x_0 > 4a + 2\eta_c$, and let there be still another, parallel, atomic layer between these two, which is subjected just to forces acting on it from the two fixed layers. Assuming these forces to be subject to the law (6. 1), it is easy to establish that in addition to the trivial equilibrium position $x_1 = 0$, the intermediate layer has two other equilibrium positions symmetric relative to the $x = 0$ plane

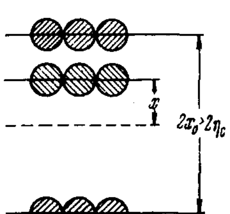

Fig. 2

$$x_2 = + x_*, \quad x_3 = - x_* \tag{6.3}$$

where x_2, x_3 are roots of the transcendental equation

$$x = (x_0 - 2a) \operatorname{th} \frac{x}{\eta_c} \tag{6.4}$$

The trivial equilibrium position is hence unstable, and the two other equilibrium positions are stable.

From these considerations it follows that an atomic layer may not simultaneously be in an interaction state corresponding to the descending portion of the $T \sim \eta$ curve with two adjacent fixed atomic layers, it must certainly be attracted to one of them. The interaction correponding to the descending branch of the $T \sim \eta$ curve may thereby not exist simultaneously at several adjacent atomic layers in practice, but can only occur between two adjacent atomic layers under the additional condition that the portions of the atomic layers between which such a state has originated are constrained in displacements. This latter can be assured only if the mentioned portions are of finite size, and surrounded by atomic layers which retain stability. The divergence of two atomic layers to a distance $\eta > \eta_c$ in some domain should be considered as the formation of a slot in the body, whose edges will again be formed by the boundaries of the body. In the mathematical solution of the problem, the mentioned sections within the body should be considered plane slits to whose lips are applied interaction forces subject to the law

$$\sigma = \sigma_c \left[1 + \frac{2v}{\eta_c}\right] e^{-2v/\eta_c} \tag{6.5}$$

where $2v$ is the distance between the lips of the slot.

The above exposition permits viewing the slit-crack theory from an unusual viewpoint, namely: considering them as nontrivial solutions of elasticity theory problems whose possibility results from the nonlinearity of the appropriate equations. Taking a nonlinear law of the form (6.1) rather than the linear law for the binding between atoms, we naturally lose the uniqueness of the solution. For example, considering the tension on a plane by forces at infinity $\sigma_{yy} = \sigma$, we shall have additional stable solutions in addition to the trivial stable solution

$$\sigma_{xx} = \sigma_{xy} = 0, \quad \sigma_{yy} = \sigma$$

and, further, their uncountable set. The formation of displacement jumps in the body at certain plane sections within the body, i.e. the formation of slots, corresponds to these solutions. The shape and size of these slots can be determined by solving the nonlinear problem formulated above. From this viewpoint the formation of a crack in an elastic body is a phenomenon of the type of buckling in the large, analogous to the snapping of a spherical shell, say. The shape and size of a dent formed during snapping are completely defined, but the place it will form is uncertain, a dent can appear at any point of a shell. The number of dents is also uncertain, in principle, several can originate.

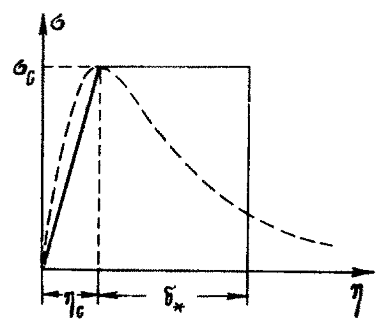

Fig. 3

An analogous uncertainty is conserved also in the problem of the equilibrium of an elastic plane, where just as in the previous example, the place of crack formation, and the quantity of cracks being formed remain uncertain.

7. The nonlinear problem formulated above, of seeking ambiguous solutions of the problem of extension of an elastic plane in which the interaction between atoms is subject to a law of the form (6.1) is quite difficult, and can hardly be solved rigorously. A successful approximate solution is given by Leonov and Panasiuk, discussed in detail in [1].

Although the mentioned authors do not give their results, the treatment which was given above by assuming that a slit exists in the body beforehand, and that its length is given in advance, nevertheless, they substantially considered the question of ambiguous solutions of the problem of the extension of a nonlinear elastic plane.

The curve of Fig. 1 is replaced in [10] by the curve in Fig. 3, according to which

Hooke's law is valid for $\eta \leqslant \eta_c$ while for $\eta > \eta_c$

$$\sigma = \begin{cases} \sigma_c & (\eta_c \leqslant \eta \leqslant \eta_c + \delta_*) \\ 0 & (\eta > \eta_c + \delta_*) \end{cases} \tag{7.1}$$

where δ_* is chosen from the requirement that the areas of the true and approximate curves be equal, i. e. from the condition $2\gamma = \sigma_c \delta_*$ (7.2)

It is easy to recognize the energy balance idea widely used in the solution of nonlinear problems in this simplification. Altough the $T \sim \eta$ graph remains nonlinear in such a simplification, nevertheless, the considered problem becomes linear since Hooke's law operates at all points within the body, and the forces applied to the slit lips are converted into a constant loading independent of the displacements of the lips.

Having been given the slit length l and requiring that the stresses at its ends be equal $\sigma_{yy} = \sigma_c$, the domain of values of l in which the posed problem has a solution, can be be determined. All the necessary comoutations are successfully carried to a conclusion, and the following fundamental results ensue.

The width of sections within the slit end on which the constant stresses $\sigma_{yy} = \sigma_c$ act is determined by the approximate expression

$$\Delta \approx \frac{\pi^2}{16} \frac{\sigma^2}{\sigma_c^2} l \tag{7.3}$$

which reduces to

$$\Delta = \frac{\pi}{4} \frac{E\gamma}{\sigma_c^2} \frac{l}{l_g} \tag{7.4}$$

if (1. 9), (2. 8) are taken into account.

The critical value of the slit length (i. e. the limiting value of this length beyond which the solution of the nonlinear problem under consideration does not exist) is obtained practically coincident with the Griffith critical length l_g (under the condition that $l \gg \delta_*$, which is always satisfied for macroscopic slots, since the quantity δ_* will be on the order of the atomic radius according to the estimate (4. 5)). Hence, (even if it is taken into account that (2. 8) lowers γ somewhat since its deduction is based on replacing the descending branch of the $T \sim \eta$ curve by a segment of a sinusoid, shown dashed in Fig. 1) Δ will be a quantity which does not exceed the atomic diameter.

The distribution of normal stresses σ_{yy} at the end of a crack $x = {}^1/_2 l + \Delta$ in the critical state, is determined by

$$\sigma_{yy}(\xi, 0) = \frac{1}{2}\sigma_c \left[1 + \frac{2}{\pi} \arcsin\left(\frac{\Delta^* - \xi}{\Delta^* + \xi}\right)\right] \tag{7.5}$$

$$\Delta^* = \frac{\Delta}{l}, \quad \xi = \frac{x - {}^1/_2 l - \Delta}{l} \tag{7.6}$$

This formula can be derived from the more awkward expression (1. 13) in [10], if it is considered that Δ^* and ξ are negligibly small quantities compared with unity. Putting $\xi = \Delta^*$ in (6. 5), we have $\sigma_{yy}(\Delta^*, 0) = {}^1/_2 \sigma_c$ (7.7)

It is hence seen that the stresses σ_{yy} decrease rapidly as a point recedes from the new end of the slit, and already become half the maximum value at distances on the order of the atomic diameter.

Therefore, according to the solution elucidated, the crack takes on a tendency to propagate when the atomic bonds at its ends are overcome on sections on the order of an atomic diameter, i. e. when two pairs of atomic arrays are uncoupled. But precisely this condition was indeed taken as the fracture criterion in deriving (2. 6). It hence becomes clear why the results of the theory in [10] agree with (2.6) and (1.9). Quantitative estimates of (4.5) and (7.4) for δ_* and Δ show that it is impossible to map the ends of the crack in the area of application of the adhesion forces as the curve of a line in a continuum (this is done in many works on cracks) since the mouth of "the beak" is $\delta_* \approx a$, and the length of "the beak" is $\Delta \approx 2a$.

In contrast to the solution obtained from the criterion (2.1), the solution considered above will be not only necessary but also sufficient since the combined work of the whole system of atoms of the body is taken into account in the neighborhood of the crack, where attention is paid to the possibility of failure of part of the elements of this system to which interaction between some atoms according to the law of the descending portion of the $T \sim \eta$ curve will correspond. In substance, the carrying capacity of a body having a slit is estimated in the present solution, while the criterion (2.1) allows only the condition of failure of the weakest elements of a system formed by a set of atoms, to be obtained. The fact that both solutions turn out to be quite close quantitatively indicates that the necessary strength condition in this problem turns out to be sufficient also, in practice. It had been impossible to forsee this in advance. For example, if (7.3) were to yield a value of Δ on the order of $10\ a$, then according to the sufficient criterion the crack length would have been several times greater than the Griffith value.

8. Under additional simplifications inherent in the linear elasticity theory, the apparatus of continuum mechanics is applied to problems in the theory of brittle fracture of bodies weakened by slits, in which: — Hooke's law is not valid, where the physical nonlinearity is quite substantial; — the deformations and angles of rotation are comparable to one; — dimensions on the order of the atomic diameter should be considered finite.

But not even this will be the principal source of error in this theory, as is the fact that plastic deformation which originate inevitably at the ends of crack in even quite brittle materials, are neglected completely.

Consequently, the formulas presented above must not be depended upon for quantitative confidence. This is precisely why the engineering theory of crack propagation has been developed primarily as a phenomenological theory. The rationality of this aspect is given the clearest foundation in the work of Irwin [11].

However, theoretical investigations based on idealized models and attempting to clarify the connection between crack propagation conditions and physical constants characterizing the body material, continue intensively. The advantage of such investigations is the deepening of insights into the comparative role of the various factors affecting the stress field in the neighborhood of the ends of the crack, and the condition of its propagation.

The reasoning expounded herein sheds light on some peculiarities of slit-crack theory and allows the following deductions.

1. The Griffith formula (1.9) can be deduced from the solution (2.2) of classical elasticity theory, and the strength criterion (1.2) in a natural extension to the case of large stress gradients (2.1).

2. The theory of slit crack propagation (in any modification) cannot manage without the physical constant of the length dimension. Such a constant will be the atomic radius in brittle fracture theory.

3. It results from (3.5) that the Griffith formula (1.9) can be considered as a result

valid not only for slit cracks, but also as an approximate formula for cracks with finite spacing between the lips under the condition that the radius ρ of the end of the crack is a quantity on the order of a.

4. The fracture condition (2.1) is not only necessary but also sufficient, judging by the fact that the resulting picture of interatomic forces in the neighborhood of the crack ends agrees with the picture which follows from the theory in [10] yielding an approximate estimate of the carrying capacity of a body weakened by a crack.

5. Slit cracks can be interpreted as the nontrivial solution of the problem of tension of an elastic plane if the connection between the stresses and strains is nonlinear, of the form of (6.1). Crack formation hence turns out to be a phenomenon of the type of buckling in the large.

BIBLIOGRAPHY

1. Panasiuk, V. V., Ultimate Equilibrium of Brittle Bodies with Cracks. Kiev, "Naukova Dumka", 1968.
2. Griffith, A. A., The phenomenon of rupture and flow in solids. Phil. Trans. Roy. Soc., London, ser. A, Vol. 221, 1920.
3. Elliot, H. A., An analysis of the conditions for rupture due to Griffith cracks. Proc. Phys. Soc., ser. A, Vol. 59, №232, 1947.
4. Sneddon, I. N., The distribution of stress in the neighborhood of a crack in elastic solids. Proc. Roy. Soc., ser. A, Vol. 187, №1009, 1946.
5. Gilman, J. J., Strength of ceramic cristals. Phys. and Chem. Ceram., N.Y.-London, Gordon and Breach, 1963.
6. Muskhelishvili, N. I., Some Fundamental Problems of Mathematical Elasticity Theory. 3rd ed., Moscow-Leningrad, Akad. Nauk SSSR Press, 1949.
7. Tetelman, A. S. and McEvily, A. J., Fracture of Structural Materials, N.Y., Wiley, 1967.
8. Cottrell, A. H., Mechanical Properties of Matter. N.Y., Wiley, 1964.
9. Leonov, M. Ia. and Panasiuk, V. V., Development of very small cracks in a solid. Prikladna Mekhanika, №4, 1959.
10. Leonov, M. Ia., Elements of the theory of brittle fracture. PMTF, №3, 1961.
11. Irwin, G. K., Fracture. Handbuch der Physik, Bd. 6, Springer, Berlin, 1958

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